

Exercise 5

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$u(x) = \frac{3}{2}x - \frac{1}{3} - \int_0^1 (x-t)u(t) dt$$

Solution

This is a Fredholm integral equation because both of the limits of integration are constant. It is of the second kind because the unknown function u appears both inside and outside the integral. It's inhomogeneous because of the $(3/2)x - 1/3$. It's not singular since neither of the limits of integration are infinite and the integrand does not become infinite in the interval of integration.